Roll No. ....

94060

B. Sc Mathematics (Hons.)

5th Semester

Examination – March, 2021

GROUPS & RINGS

Paper : BHM-352

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section - V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) Show that union of two subgroups in a subgroup if and only if one is contained in other. 6

(b) Show that every subgroup of a Cyclic group is cyclic. 6

2. (a) Show that the order of every element of a finite group is a divisor of the order of the group. 6

(b) If H is the only subgroup of finite order in the group G, then prove that H is the normal subgroup of G. 6

SECTION – II

3. (a) Show that every homomorphic image of a group G is isomorphic to some quotient group of G. 6

(b) If a is a fixed element of a group G then show that the mapping $T_a : G \to G$ defined as $T_a(x) = a^{-1}xa$ in an automorphism of G. 6

4. (a) If p is a prime number and G is non-abelian group of order p^3 then show that Z(G) has exactly p elements. 6

(b) Show that every group is Isomorphic to a permutation group. 6

SECTION – III

5. (a) Show that the set $(\{0, 1, 2, 3, 4\}, +_5, \times_5)$ is a field. 6

(b) If R is a Commutative ring with unity and has no proper ideals then show that R is a field. 6

6. (a) If S and T are two ideals of a ring R, then show that $(S+T)/S \cong T/(S \cap T)$. 6

(b) If an ideal S of a commutative ring R with unity in maximal then show that R/S is a field. 6

SECTION – IV

7. (a) If R is an Euclidean ring and S be an ideal of R then show that there exist an element $a_0 \in S$ such that $S = \{a_0 r : r \in R\}$. 6

(b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 6

8. (a) Show that the polynomial ring Z[x] over the ring of integer is not a principal ideal ring. 6

(b) Show that the polynomial $x^4 + 1$ is irreducible over Q. 6

SECTION – V

9. (a) If G = {0, 1, 2, 3, 4, 5} is a group under the binary operation addition modulo 6, find the order of elements 2 and 4. 2

(b) How many generators are there of the cyclic group G of order 8 ? 2

(c) Use Lagrange's theorem to show that any group of prime order can have no proper subgroup. 2

(d) Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ as the product of disjoint cycles. 2

(e) If S is an ideal of a ring R with unity such that $1 \in S$ then show that $S = R$. 2

(f) Show that every homomorphic image of a commutative ring in commutative. 2
